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A CONVERGENT SHOCK WAVE IN A HEAT CONDUCTING GAS

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A convergent spherical wave is intensified to infinity as it approaches the center, as determined by Guderley [1] and L.D.Landau and by K.P.Stanyukovich [2] who proposed an automatic model solution of a pure gas-dynamics problem relating to the focusing of a wave. However, in the course of a similar physical occurence we can expect a strong effect of heat conductivity, since the resulting high temperature gives rise to a radiant heat exchange.

It is worthwhile to examine the focusing of a wave, taking into account the above factor, and in particular to determine whether a cumulation of unlimited energy can also take place under this premise.

For simplicity we will analyze the case of a wave of a power only sufficient to produce a balance of radiation and matter at all points, in which the width of the heat zone in large compared to the range of radiation, and only during the wave's early stage, when its width is still smaller than its radius.

Heat conductivity erodes the wave front: a zone of heat and of gas movement is formed ahead of the advancing consolidation. The general features of this effect are described in a book by Ya.B.Zel'dovich and U.P.Royzer [3] .

We will first calculate the width of the heat zone for a flat stationary wave and for this purpose will write the conservation equations for a wave moving through a cold gas:

$$\rho u = \rho_0 D, \qquad p + \rho u^2 = \rho_0 D^2$$

$$\rho u \left(\frac{u^2}{2} + \frac{1}{\gamma - 1} \frac{p}{\rho} \right) + pu + Q = \frac{\rho_0 D^3}{2}$$

Here Q - the flow of heat, the remaining symbols are conventional ones (the wave lies within the adopted system of coordinates).

Let us examine the case when the radiant energy participates only in the heat exchange, but its density $(4/c)\,\sigma T^4$ is still low, as compared to $p/(\gamma - 1)$, that is, we will be using the ideal gas state equation $p = (R/\mu)\rho T$, neglecting the pressure of radiation.

For radiant conductivity

$$Q = -\frac{lc}{3} \frac{d}{dx} \left(\frac{4}{c} \sigma T^4 \right)$$

where c - velocity of light, l - distance covered by radiation.

If this is determined by comptonic diffusion, then

$$l \sim \rho^{-1} \quad \text{or} \quad l = L\,(\rho_0 / \rho) \qquad (L = \text{const})$$

Thus, we have four equations, using which we can express u, p, ρ and dT / dx in terms of T.

Omitting the computations, we present the final result for magnitudes ρ and dT / dx:

$$\frac{\rho}{\rho_0} = 2 \left[1 + \left(1 - \frac{8(\gamma - 1)}{(\gamma + 1)^2} \tau \right)^{1/2} \right]^{-1}$$

$$\left(\tau = \frac{T}{T_k}, \quad T_k = \frac{2(\gamma - 1)}{(\gamma + 1)^2} \frac{\mu D^2}{R} \right) \qquad (1)$$

Here T_k - the ultimate temperature behind the wave;

$$\frac{d\tau}{dx} = \ldots \quad \text{[illegible]}$$

$$\ldots \text{[illegible]} \quad (2)$$

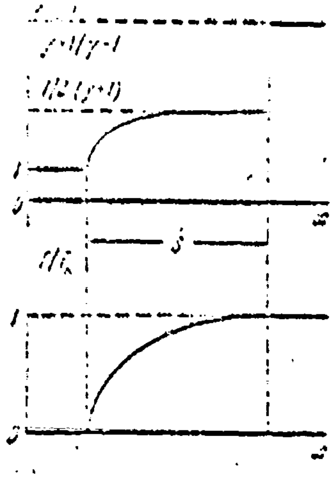

Fig. 1.

In the heated zone τ varies from 0 to 1, at the same time ρ/ρ_0 increases from 1 to a value $\frac{1}{2}(\gamma+1)$ (as seen from equation (1) and proceeds to surge to an ultimate value $(\gamma+1)/(\gamma-1)$ where $(\gamma \leq 3)$. Integrating equation (2) with respect to τ from 0 to 1, we find the full width δ of the heated zone before the surge of consolidation

$$\delta = DJ, \quad J = \int_0^1 \frac{\left(1+\sqrt{1-8\tau(\gamma-1)/(\gamma+1)^2}\right)\tau^{3/2}d\tau}{2\tau/(\gamma+1) - 1 + \frac{1}{2}\left(1+\sqrt{1-8\tau(\gamma-1)/(\gamma+1)^2}\right)}$$

With $\gamma = 1.4$

$$J = 0.871, \qquad \delta = 1.73\cdot 10^{-3}\,\frac{L\rho_0^{1}D^{5}}{p_0R^{5}} \qquad (3).$$

Schemes of spatial distribution of ρ and T in the heated zone are shown in Fig. 1.

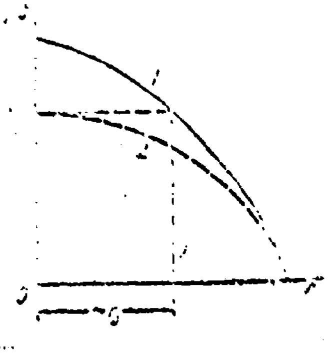

Fig. 2.

Let us return to the converging wave. As seen from equation (3), the width of the heated zone increases rapidly with an increase in D, that is, with the progress of the wave toward the center. Arrival of the heat wave front at the center will be the precise moment when temperature ceases to increase

gradually and the temperature attained by that time will be in the order of a maximum for the entire process.

In the converging wave $D = A / r^{\alpha}$ ($\alpha = 0.395$ for $\gamma = 1.4$), where A characterizes the power of the wave (its velocity on a unit radius).

Let us determine the characteristic dimension of the wave r_0, assuming that S is approximately r_0. Substituting the expression for D into equation (3) and taking $S \sim r_0$, we obtain the relation for r_0, from which we find

$$r_0 \sim \left(\frac{L\mu [illegible] A^2}{\rho_0 [illegible]}\right)^{\frac{1}{1+3\alpha}}$$

Now let us determine the maximum temperature of the process. In the shock wave $T \sim D^2 / R$, which means that in a converging wave

$$T \sim \mu A^2 / R\, r^{2\alpha}$$

Substituting r_0 for r, we obtain an expression for the maximum temperature

$$T_{max} = \text{const}\, (\rho_0^{2\alpha} [illegible] L^{-2\alpha} [illegible])^{\frac{1}{1+3\alpha}}$$

Specifically, for $\gamma = 1.4$ ($\alpha = 0.395$)

$$T_{max} = \text{const}\, \rho_0^{[illegible]} [illegible] (L\sigma)^{[illegible]}$$

In which const - a dimensionless coefficient, approximately unity; it can be found only through a complete solution of the problem relating to the focusing of a heat-conducting wave, which, in principle, can be accomplished numerically.

And so, in the presence of heat conductivity, the temperature attained is limited, but because of the intensification of the wave (increment A), it can be made as great as desired. In that sense, limitation of temperature by heat conductivity is not mandatory.

The scheme of focusing of a wave in the case under consideration is illustrated by sketch.

The heat wave and shock wave reach the center in that order. Each one of them is described near the center by their self-modeling solution (which we will not discuss), but there is no combined self-modeling solution for the entire process.

Let us analyze qualitatively the behavior of the shock wave near the center.

During its focusing stage the temperature remains constant, near the center it does not depend on r, nor on t, in other words, the wave is isothermal. Its amplitude could move toward zero, an ultimate limit, or toward infinity (disregarding oscillations). We will demonstrate that a third possibility is actually realized.

Tendency toward zero can be excluded, since we know that any weak shock wave does not become weaker near its center, but becomes intensified according to the law

$$\Delta p \sim r^{-1}$$

If the amplitude strove toward an ultimate limit, the wave near its center would be described by a self-modeling solution of constant amplitude. An attempt to substitute such a solution into the equations of motion reveals the fact that it does not satisfy them. Only the third possibility remains, namely, that of unlimited increase (the law governing this increase has not been determined). And so, heat conductivity has only modified the unlimited cumulation, but has not eliminated its

in place of a limited density and an unlimited temperature we now have an ultimate temperature and an infinite density.

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